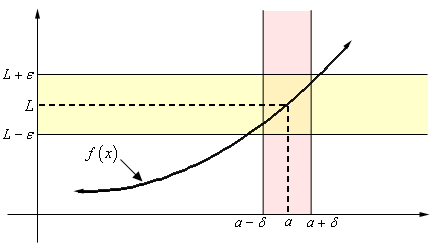
**Limit & Continuity**

***Introduction:*** In this chapter we will study about limit that is the core tool of calculus and all other calculus concepts are based on it. A function can be undefined at a point, but we can think about what the function "approaches" as it gets closer and closer to that point (this is the "limit"). Also the function may be defined at a point, but it may approach a different limit. There are many, many times where the functional value is the same as the limit at a point. Limit is used to define continuity, derivative and integral of a function.

***Limit of a function****:* The number is called limit of a function at a point if approaches closer and closer to from both sides and consequently approaches closer and closer to Symbolically it is written as,

.

***Graphical representation of “limit of a function” at a point:*** Let the function be . If , then by the definition of limit we have, if for each given , there exists a positive number (depending on ) such that , whenever  i.e. whenever , i.e. whenever , but .



The point  of the graph of the function  lies between the two lines  and  provided that  lies in the interval , . This implies that as long as  belongs to the interval , , the graph of the function  lies within the rectangle bounded by straight lines , ,  and , where  is the centre of the rectangle. Here  can be chosen as small as we wish such that the rectangle can be made to have as small as an altitude  as we wish. Thus we can explain shortly,  exists nearer the point .

***Mathematical* or  *definition of limit of a function:*** The number is called limit of a function at approaches if for any given positive number , we can find another positive number such that , for all values of *x* satisfying

.

Symbolically it is written as,

**Left Hand Limit:** If the values of can be made as close as we like to by taking values of *x* sufficiently close to (but less than) then we write,

**Right Hand Limit:** If the values of can be made as close as we like to by taking values of *x* sufficiently close to (but greater than) then we write,

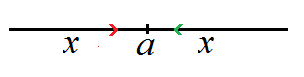
***Existence of limit of a function at :***

The limit of a function at that isexists if

1. *.*

***Fundamental Properties of limit:***

If are two functions and

***Change of limit of a variable*:**

**Left hand limit:**

Let and when.

Now, .

**Right hand limit:**

Let and when.

Now, .

**Theorem-01:** If  exists then it must be unique.

**Proof:** Let,  and .

Now we have to show that .

Let , then there exists , where  whenever  and  whenever .

Now 









This is evidently false. Hence ** . (proved)**

**Theorem-02:** If the two functions  and  are defined in the neighborhood of a point  by  and  then show that .

**Proof:** Since,  implies that if for each given , there exists a positive number such that , whenever  

Again as  implies that if for each given , there exists a positive number such that , whenever  

Let, , then from (1) and (2) we have

 and  whenever .

Now 







So by the definition we have

.  **(Showed)**

**Theorem-03:** If the two functions  and  are defined in the neighborhood of a point  by  and  then show that .

**Proof:** Since,  implies that if for each given , there exists a positive number such that , whenever  

Again as  implies that if for each given , there exists a positive number such that , whenever  

Let, , then from (1) and (2) we have

 and  whenever .

Now 







So by the definition we have

.  **(Showed)**

***Problem-01:*** A function is defined as follows:



Does exist?

***Solution*:** Given that,

Since . Sodoes not exist.

***Problem-02:*** A function is defined as follows:



Find the value of .

***Solution*:** Given that,



Since . So exists.

The limiting value is,

.

***Problem-03:*** If then find limits from the left and the right of . Does the limit of at exist?

***Solution*:** Given that,

Here,  and  both are exist but they are not same.

i.e, . Sodoes not exist.

***Problem-04:*** If then find limits from the left and the right of . Does the limit of atexist ?

***Solution*:** Given that,

Here,  and  both are exist but they are not same. i.e, . Sodoes not exist.

***Problem-05:*** A function is defined as follows:



Discuss the existence of .

***Solution*:** Given that,



Here,  and  both are exist but they are not same.

i.e, . Sodoes not exist.

***Homework*:**

***Problem-01*:** A function is defined as follows:



Find the value of .

***Problem-02*:** A function is defined as follows:



Find the value of .

***Problem-03:*** If then find limits from the left and the right of . Does the limit of atexist ?

***Problem-04:*** If then find limits from the left and the right of . Does the limit of atexist ?

***Problem-05:*** If then show that does not exist but exists.

***Problem-06:*** If then find limits from the left and the right of . Does the limit of at exist?

***Some important limits:***

1.  **2.**

**Proof:** Given that, **Proof:** Given that,



1.  **4.**

**Proof:** Given that, **Proof:** Given that,



1. 

**Proof:** Given that,



**L’ Hospital’s Rule:** If two functions and are continuous at  , also their derivatives , are continuous at this point and  but then L’ Hospital’s rule states as,



In case,, the rule maybe extended.

**Indeterminate forms:** If then it is called an indeterminate form at . The forms , , , ,  and  are also indeterminate forms.

**Evaluate the following limits:**

**Problem 01: Find** **Problem 02: Find**

**Sol:** Given that, **Sol:** Given that,



**Problem 03: Find**  **Problem 04: Find**

**Sol:** Given that, **Sol:** Given that,



**Problem 05: Find**  **Problem 05: Find**

**Sol:** Given that, **Sol:** Given that,



**Problem 06: Find**  **Problem 07: Find**

**Sol:** Given that, **Sol:** Given that,



**Problem 08: Find**  **Problem 09: Find**

**Sol:** Given that, **Sol:** Given that,



**Homework:**

**Problem 01: Find**Ans: 1

**Problem 02: Find**Ans: 

**Problem 03: Find**Ans: 

**Problem 04: Find**Ans: 

**Problem 05: Find**Ans: 

**Problem 06: Find**Ans: 1

**Continuity:** A functionis said to be continuous at a point provided the following three conditions are satisfied:

1. exists,
2. is defined,
3. .

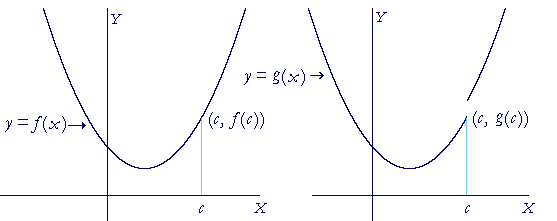
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Fig. (a) Continuous function. Fig. (b) Discontinuous function.

If one or more of the conditions of this definition fails to hold, then the function is discontinuous at .

**Distinguish between functional value and limiting value:** If  be a function then the functional value and the limiting value at  are  and respectively. The statement  stands for the value of  when  approaches closer and closer to  except. In this case we do not care to know what happens when  is put equal to . But the statement  stands for the value of  when  is exactly equal to , obtained by substituting of  for  in the expression , when it exists.

***Problem-01:*** A function is defined as follows:



Discus the continuity at .

***Solution*:** Given that,

Here,. So exists and the limiting value is,

.

Now, the functional value atis,



Since,, the given function is continuous at .

***Problem-02:*** Test the continuity of the functionat the point 

***Solution*:** The given function is,





Here,. So exists and the limiting value is,

.

Now, the functional value atis,





Since,, the given function is continuous at .

***Problem-03:*** If for what value of *a* ,is continuous at 

***Solution*:** Given that,

And, the functional value atis,





Now, the given function will be continuous at ,

if







 (*Ans*.)

***Problem-04:*** If then test the continuity at 

***Solution*:** Given that,











.

And, the functional value atis,



Since,, the given function is continuous at .

***Problem-05:*** If then test the continuity at 

***Solution*:** Given that,













.

And, the functional value atis,



Since,, the given function is continuous at .

***Problem-06:*** A function is defined as follows:



Discus the continuity at .

***Solution*:** Given that,

Here,. Sodoes not exist.

Hence, the given function is discontinuous at .

**Problem-07:** Using the  definition of limit to show that .

***Solution*:** Here, and .

We must show that for any given positive number , we can find a positive number  such that , whenever 

i.e. , whenever .

Now 







 where 

Hence, we have shown that .

**Problem-08:** Using the  definition of limit to show that .

***Solution*:** Here, and .

We must show that for any given positive number , we can find a positive number  such that , whenever 

i.e. , whenever .

Now 







 where 

Hence, we have shown that .

**Homework:**

**Problem-01:** Using the  definition of limit to show that .

**Problem-02:** Using the  definition of limit to show that .

***Problem-03*:** A function is defined as follows:



Test the continuity at .

***Problem-04*:** Discuss the continuity of the functionat the point 

***Problem-05:*** Test the continuity of the functionat the point 

***Problem-06:*** Find a non-zero value for the constant *k* that makes continuous at.

***Problem-07:*** If then test the continuity at 

***Problem-08:*** If then test the continuity at 

***Problem-09:*** If then test the continuity at 

**Problem-09:** Using the  definition of limit to show that .

***Solution*:** Here, and .

We must show that for any given positive number , we can find a positive number  such that , whenever 

i.e. , whenever .

Now 





 where 

Hence, we have shown that .

**Problem-10:** Using the  definition of limit to show that .

***Solution*:** Here, and .

We must show that for any given positive number , we can find a positive number  such that , whenever 

i.e. , whenever .

Now 





 where 

Hence, we have shown that .